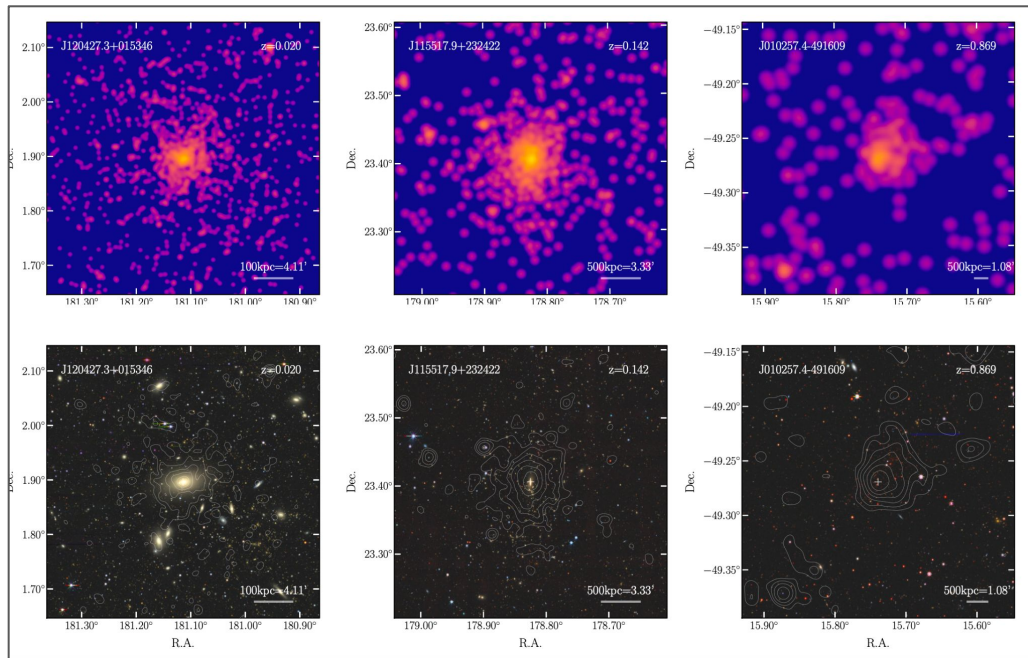


# Modeling Population(s) of Galaxy Clusters

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First Results from the SRG/eROSITA All-Sky  
Survey: From Stars to Cosmology  
Garching, Germany  
18 September 2024



Bulbul+ 2024, A&A

*Congratulations to Esra and the eRosita cluster team for  
all their efforts to publish the eRASS1 cluster catalog!*

# From the archives (literally...)

Image of a transparency  
from a 1998 talk

ROSAT observations had  
measured Coma's  
baryon fraction  
Briel, Henry + Bohringer 1992

How well would cluster  
gas trace the cosmic  
ratio? *Need Upsilon-bar!*

## the clustered mass density $\Omega_m$ from nucleosynthesis and the cluster baryon fraction

simple argument -

White et al '93

- 1) primordial nucleosynthesis constrains  
baryon density  $\Omega_b h^2$
- 2) mean baryon fraction in rich clusters

$$\bar{f}_b(\delta_c) = \bar{\Upsilon}(\delta_c) \frac{\Omega_b}{\Omega_m}$$

mean interior density contrast  $\delta_c \equiv \frac{\bar{\rho}(<r_{\delta_c})}{\rho_c}$

is natural scale variable for gravity and

independent of  $H_0$  (easy to apply to obs)

- 3) measure  $\bar{f}_b(\delta_c)$ , calibrate  $\bar{\Upsilon}(\delta_c)$ , then

$$\Omega_m = \bar{\Upsilon}(\delta_c) \frac{\Omega_b}{\bar{f}_b(\delta_c)}$$

# From the archives...

Image of a transparency  
from a 1998 talk

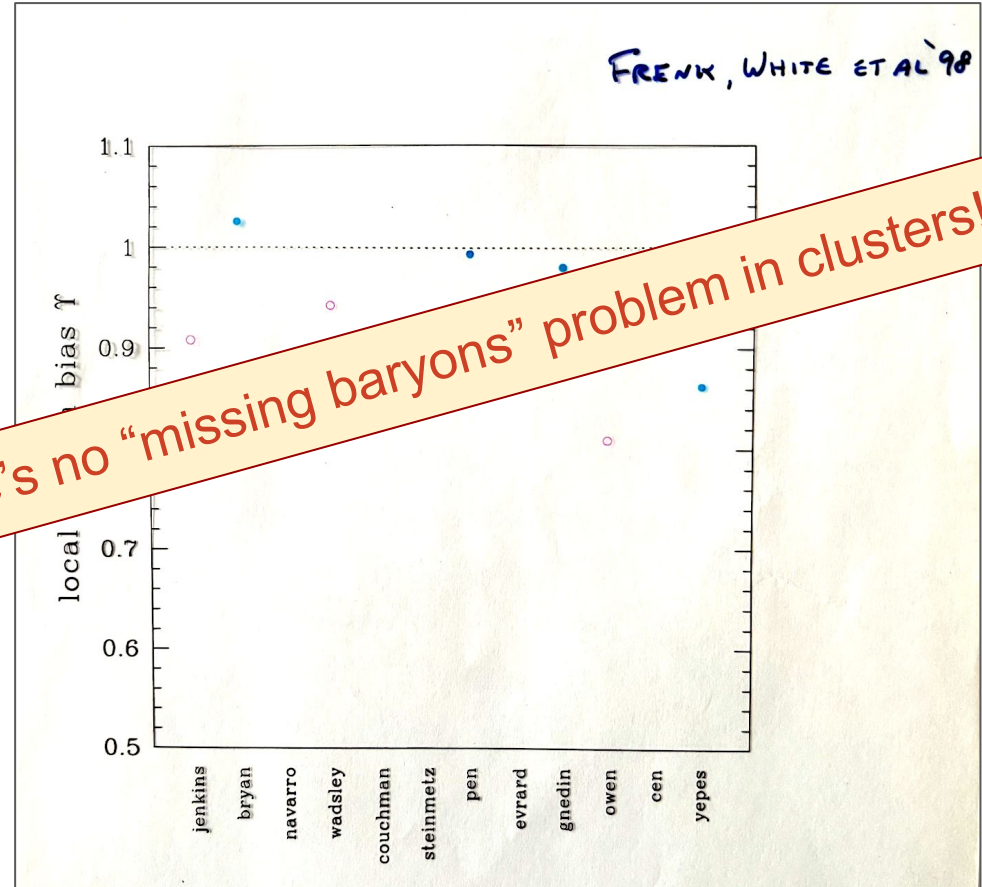
Multiple simulators had  
run the reference

“Santa Barbara cluster”

Frenk et al 1999

Local baryon fraction not  
strongly biased wrt the  
cosmic value

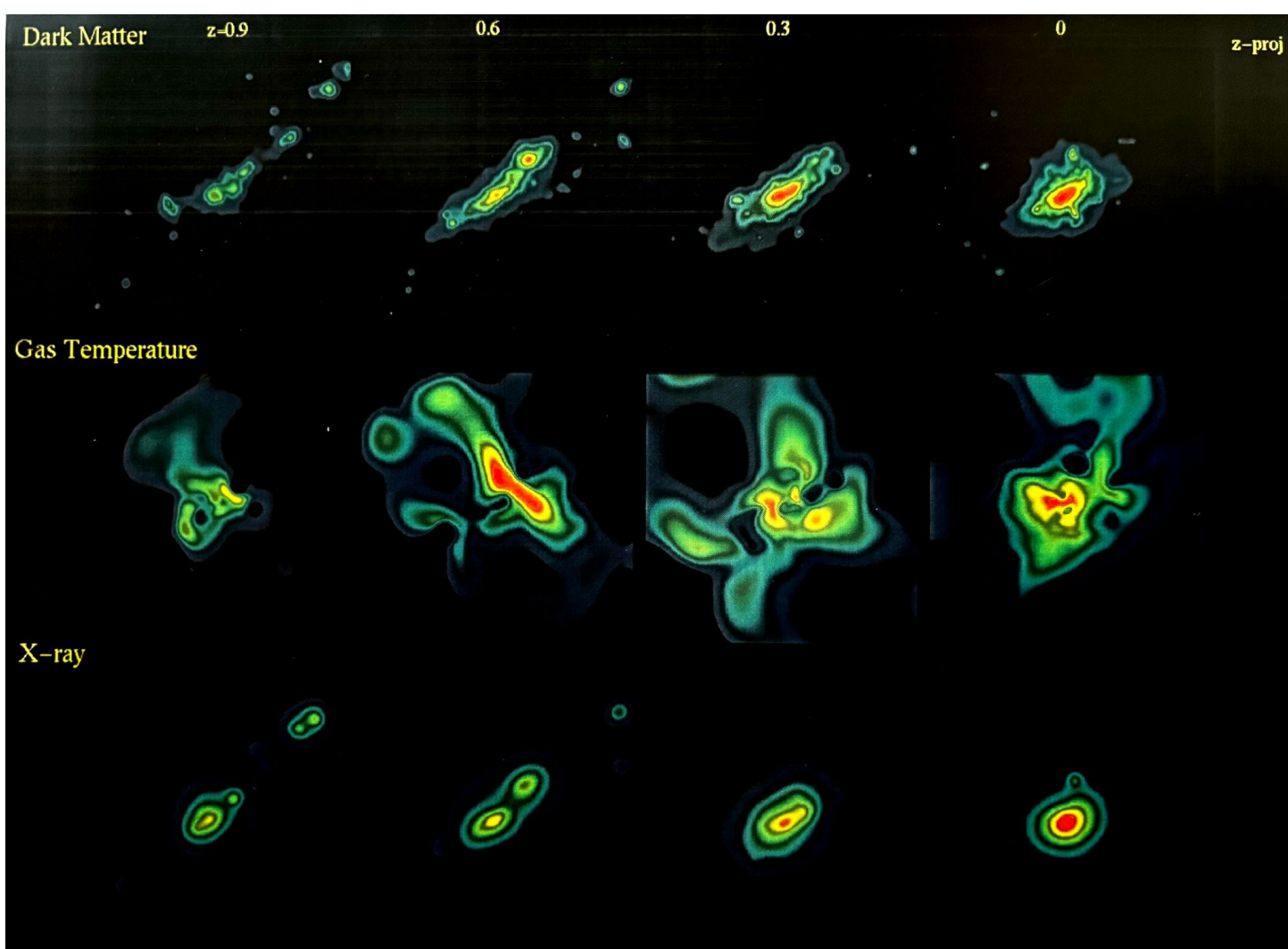
Upsilon-bar  $\sim 1$



# From the archives...

Insights into component dynamics and thermal structure

Evrard & Gioia 1998



# **The Galaxy Cluster – Massive Halo Connection**

# Galaxy Clusters at the crossroads (2011 KITP image)

## *Cosmology:*

$$\Omega_m - \sigma_8$$

gravity,  $D(z)$

GR extensions

SIDM

$$f_{nl}$$

+ complementarity

## *Astrophysics:*

most massive galaxies

(+ descendents of first galaxies)

most massive SMBHs

plasma processes

chemistry

galaxy mergers

strong lensing of high-z

structures

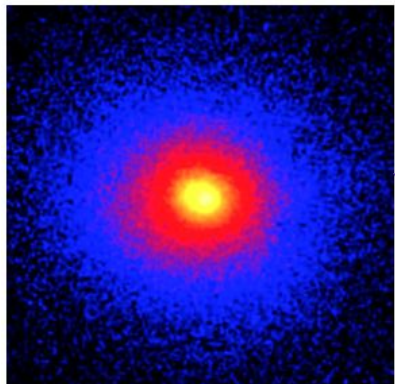
+ lots more!



Background image credit:

Gustavo Yepes (Universidad Autonoma Madrid)  
and MultiDark collaboration.

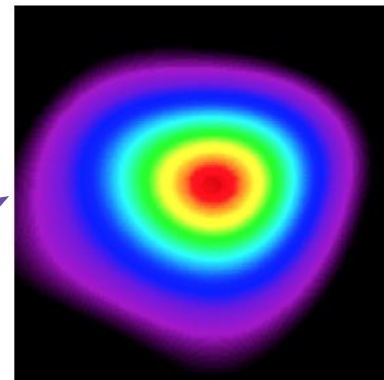
# Clusters are massive, multicomponent cosmic halos



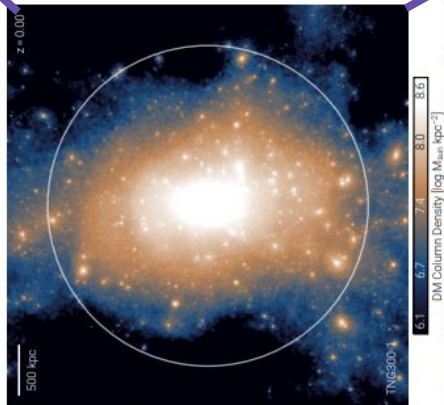
**X-ray flux**  
from hot gas  
bremsstrahlung  
best for identifying  
clusters (least projection)



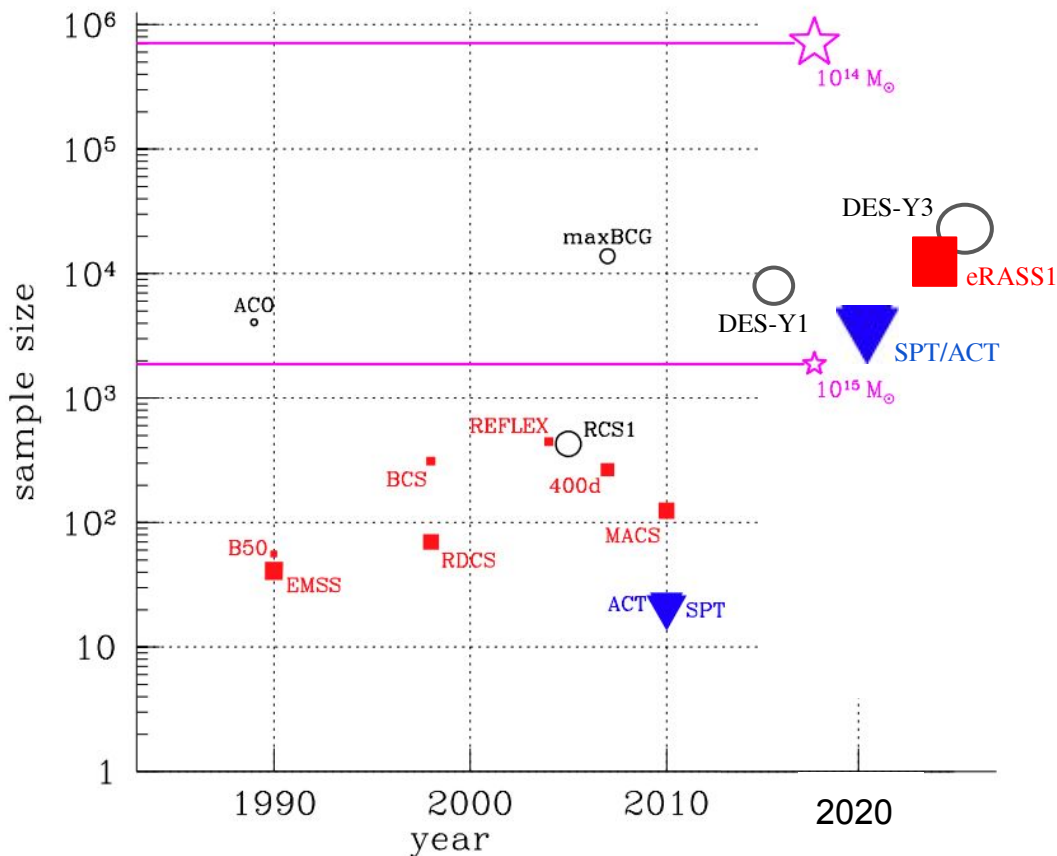
**Stars**  
redshifts!



**SZ effect** from  
upscattering of  
CMB photons  
extends to high-z  
easily



“dark matter” halo



Allen, Evrard, Mantz 2011, ARAA

## Galaxy Cluster/Halo catalogs:

Optical/IR

X-ray

Sunyaev-Zel'dovich (SZ)

Theory (all sky,  $M_{200m}$ )

- Increasing size and depth using both hot gas and galaxies as tracers.
- Growing **spectroscopic coverage**
- $z > 1$  is no longer the frontier  
Spitzer: SPARCS, GOGREEN, +
- $z > \sim 3$  is now the frontier  
Protoclusters: MAHALO, BOSS, +

**Key to cluster science: exploit overlapping, multi-wavelength surveys**



**Population Model Ingredients:  
Halo mass function (HMF) &  
Scaling Relations (MPRs/MORs)**

# Population modeling I. Halo Mass Function (HMF)

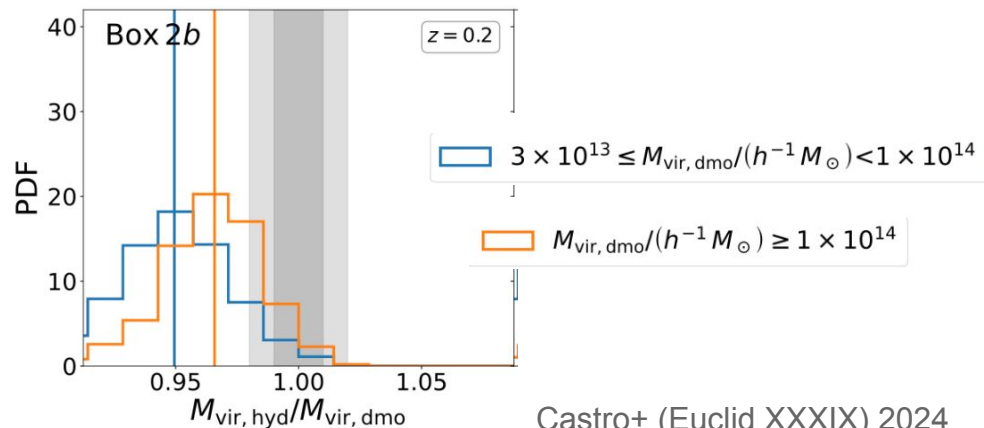
Press & Schechter 1974	<b>Analytic form</b> + 1e3 particle <b>N-body</b> ( <b>4600+ citations!</b> )
Jenkins+ (Virgo Cons.) 2001	<i>Hubble Volume</i> (1e9) and other <b>N-body</b> → <b>fit parameters</b> ( <b>1500</b> )
Tinker+ 2008	Multiple 1e9 <b>N-body</b> → <b>fit parameters (+ ext.)</b> ( <b>1600</b> )
Bocquet+ 2020	Multiple 1e12 <b>N-body</b> → <b>emulator</b> ( <b>~100</b> )
Castro+ (Euclid XXIV) 2023	Multiple 1e12 <b>N-body</b> → <b>fit parameters</b> ( <b>~30</b> )

Precise calibrations must be weighed against this:

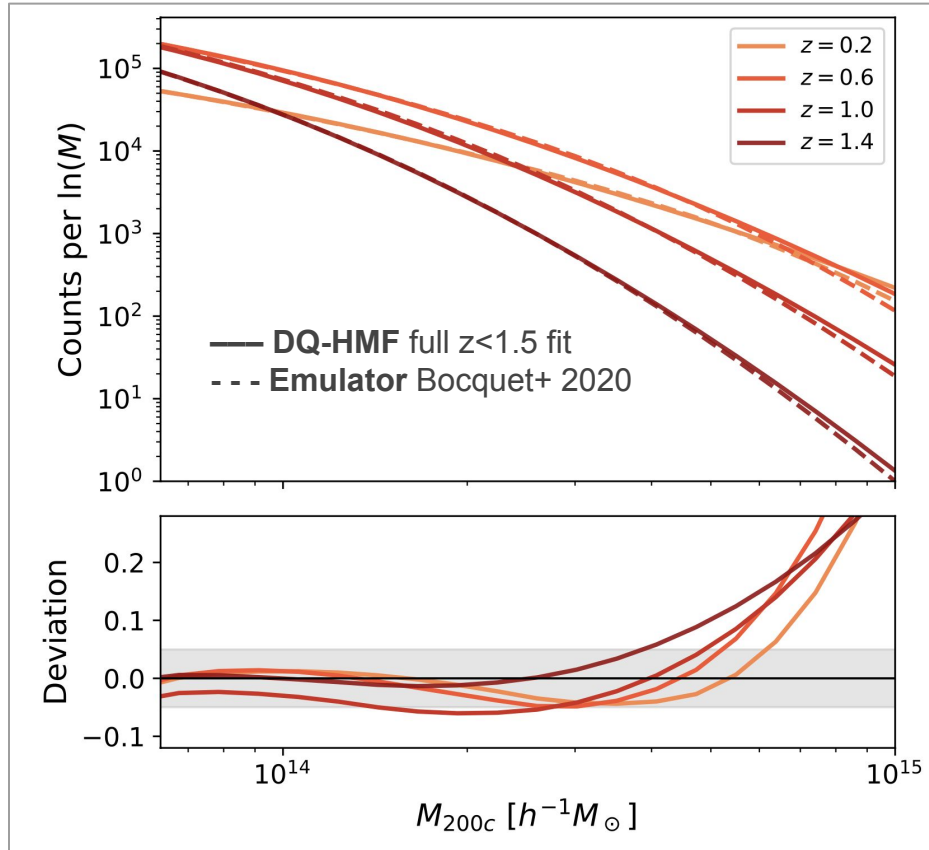
***DM-only universes are not ours!***

Galaxy formation drives **baryon loss**, reducing total halo mass.

A “**world estimate**” requires multiple, independent simulations.



# Modest alternative: simple, dual-quadratic form (DQ-HMF)



Define mass relative to a pivot value

$$\mu \equiv \ln(M/M_p)$$

Expand log of space density as a quadratic

$$\ln \left[ \frac{dn(\mu, z)}{d\mu} \right] = - \sum_{i=0}^2 \frac{1}{i!} \beta_i(z) \mu^i$$

Normalization and slope at the pivot-mass behave quadratically with  $z$

$$\beta_i(z) = \beta_{i,n} + \beta_{i,z} (z - z_p) + \frac{1}{2} \beta_{i,z^2} (z - z_p)^2 \quad ; \quad i \in \{0, 1\},$$

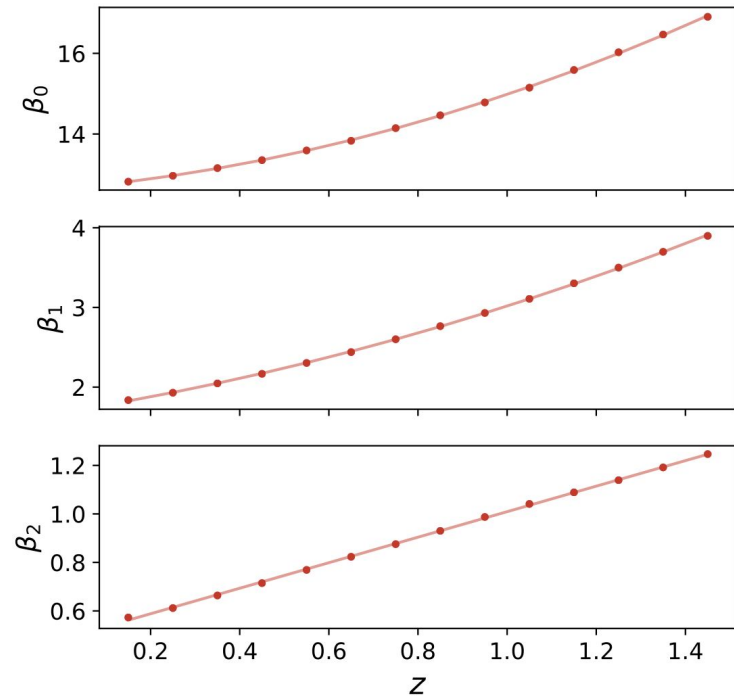
while curvature is linear.

$$\beta_2(z) = \beta_{2,n} + \beta_{2,z} (z - z_p).$$

# DQ-HMF benefits from simplicity

- Eight, directly interpretable parameters
- Computationally efficient for MCMC chains
- Convolution with log-normal mass-observable relation is **analytic**

Parameter	Definition
$\beta_i(z)$	HMF evolving shape in $\mu$ , $i \in [0, 2]$
$\beta_{i,n}$	normalization of $\beta_i$ at $z_p$
$\beta_{i,z}$	redshift gradient of $\beta_i$ at $z_p$
$\beta_{i,z^2}$	redshift curvature of $\beta_i$ at $z_p$
$M_p$	pivot mass
$z_p$	pivot redshift



**Figure 2.** DQ-HMF model parameters,  $\beta_i(z)$ , derived from fitting the Mira-Titan HMF at the redshifts shown as points. Lines show the fits to equations (2) and (3), with parameter values given in Table 2.1.

# Population modeling II. Scaling Relations

## Intrinsic **halo** scaling: Mass-Property Relations (MPRs)

- Available from “full-physics” simulations of large cosmic volumes (or “zooms”)
  - Constant power law (PL) forms are rare, but **running PL in  $\{M, z\}$**  ok
  - Scatter is **log-normal** to first order, skew-normal for some properties
- 

## Observed **cluster** scaling: Mass-Observable Relations (MORs)

- Build from light-cone outputs of “full-physics” simulations (or N-body + baryon pasting)
- **Projection often adds skew to  $\text{Pr}(\text{obs} \mid M, z)$** , less so for X-ray flux!
- Mis-centering must be modeled for optical (& SZ) clusters
- Not generic: form of  $\text{Pr}(\text{obs} \mid M, z)$  must be tailored to a survey/selection specifics

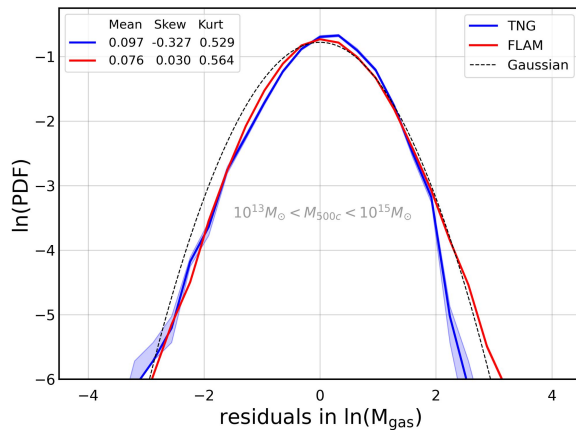
# Evidence for log-normal MPRs

Illustris-TNG&Cluster + FLAMINGO sims: **normed residuals from KLLR mean**

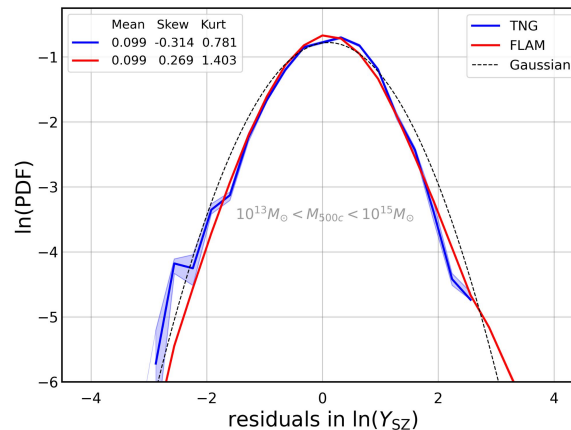


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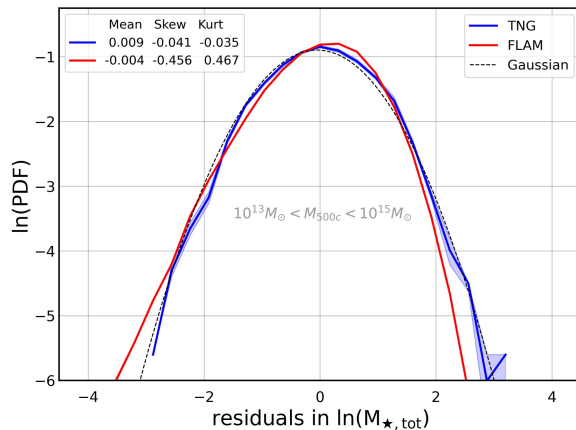
$M_{\text{gas}}$



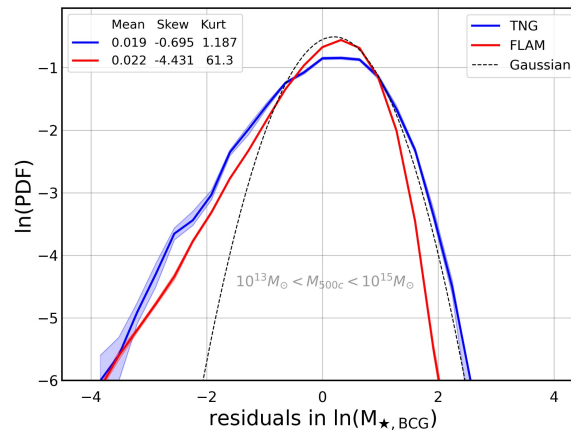
$Y_{\text{SZ}}$



$M_{\text{star,tot}}$



$M_{\text{star,BCG}}$



# DQ-HMF & PL + log-normal MPR $\rightarrow$ *analytic forms*

## Log-normal property kernel

$s = \ln(\text{property})$  in appropriate units

$$P(s|\mu) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{[s - \langle s \rangle(\mu)]^2}{2\sigma^2}\right\},$$

After convolution, find

$$\langle s \rangle(\mu) = \varpi + \alpha \mu. \quad \text{power law mean}$$

Space density

$$\ln\left[\frac{dn(s, z)}{ds}\right] = \ln A - \beta_0(z) - \frac{\beta_2(z)(s - \varpi)^2 + 2\alpha\beta_1(z)(s - \varpi) - \beta_1^2(z)\sigma^2}{2(\alpha^2 + \beta_2(z)\sigma^2)},$$

Mean mass  
selected by  
property

$$\ln\langle M | s, z \rangle = \frac{(s - \varpi)/\alpha - (\beta_1(z) - 1/2)\sigma^2/\alpha^2}{1 + \beta_2(z)\sigma^2/\alpha^2}.$$

$$A = \frac{1}{\sqrt{\alpha^2 + \beta_2(z)\sigma^2}}.$$

Explicit mixing of  
HMF shape and  
MPR parameters  
 $\rightarrow$  **cosmo/astro  
degeneracies**

**Which *Intrinsic* Properties are the Best Proxies for Total Halo Mass?**



# Mass Proxy Quality of IllustrisTNG & FLAMINGO halos

**Focus: Scatter in true halo mass given one or more properties.**

Explicit mass dependence via KLLR on halo populations at discrete redshift.



Eddie Aljamal

$$\sigma_{M_{500c} | s}^2 = \left( \alpha(\mu, z)^T \mathbf{C}^{-1}(\mu, z) \alpha(\mu, z) \right)^{-1}$$

**Steeper scalings** (higher alpha) or (and?) **low intrinsic scatter** are best, and strong anti-correlations.

**Table 3.** Sample sizes for halos with  $M_{500c} > 10^{13} M_{\odot}$

Redshift	TNG300-1	TNG-Cluster	FLAM-L1_m8
2	299	318	10 288
1.0/1.05 <sup>a</sup>	1 290	351	42 337
0.5	2 015	352	71 171
0	2 548	341	91 242

**Table 1.** Definition of gas properties used in this work.

Property	Definition
$M_{\text{gas}}$	Total hot gas mass ( $T > 10^5$ K) within $R_{500c}$
$T_{\text{sl}}$	Core-excised spectroscopic-like temperature for gas with $k_B T \geq 0.1$ keV and $0.15R_{500c} < r < R_{500c}$ , eq. (2)
$T_{\text{mw}}$	Mass-weighted average temperature of hot gas within $R_{500c}$ , eq. (3)
$Y_X$	X-ray pressure of gas in $R_{500c}$ , eq. (9)
$Y_{\text{SZ}}$	tSZ pressure of gas in $R_{500c}$ , eq. (8)

# Hot gas scaling relations: gas mass & temp. slopes



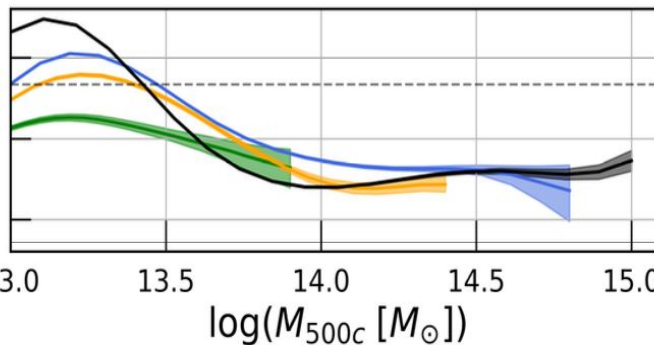
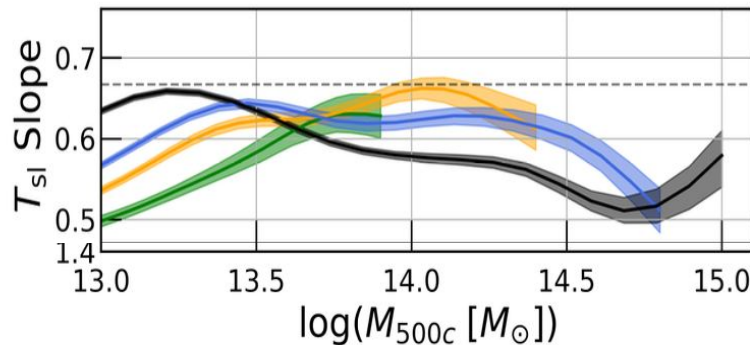
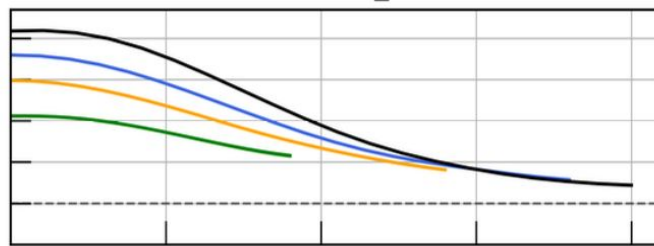
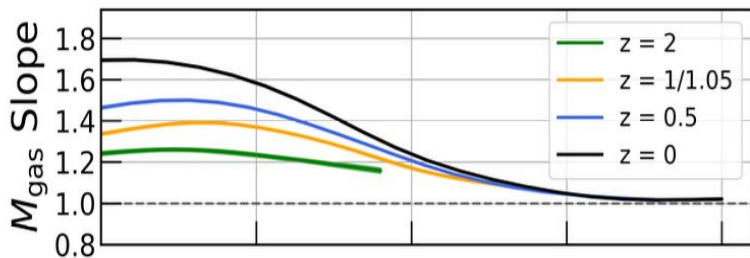
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Robustness to numerical method



TNG300 + TNG-Cluster

FLAM-L1\_m8



- Approaches self-similar value (1)
- Group slope steepens with time

- Cluster value consistently below self-similar (2/3)
- Weak redshift dependence

# Hot gas scaling relations: pressure (2 types) slopes



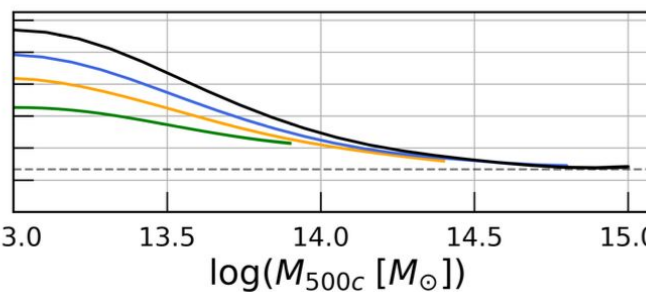
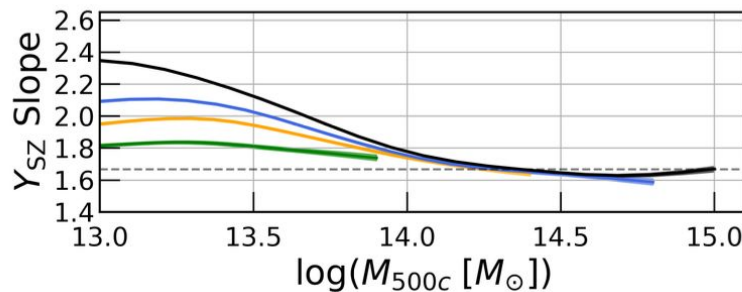
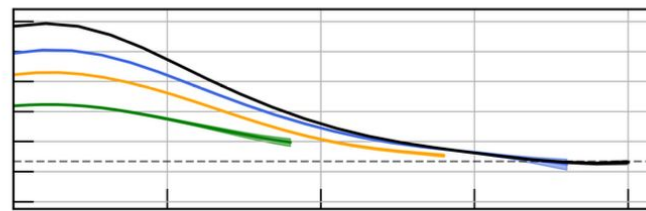
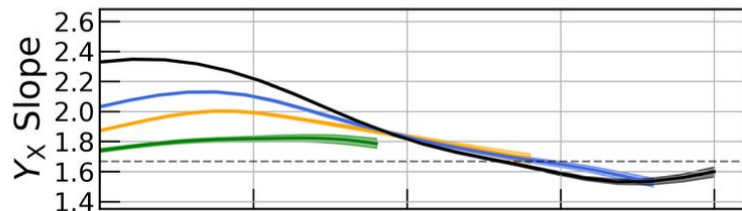
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Robustness to numerical method



TNG300 + TNG-Cluster

FLAM-L1\_m8



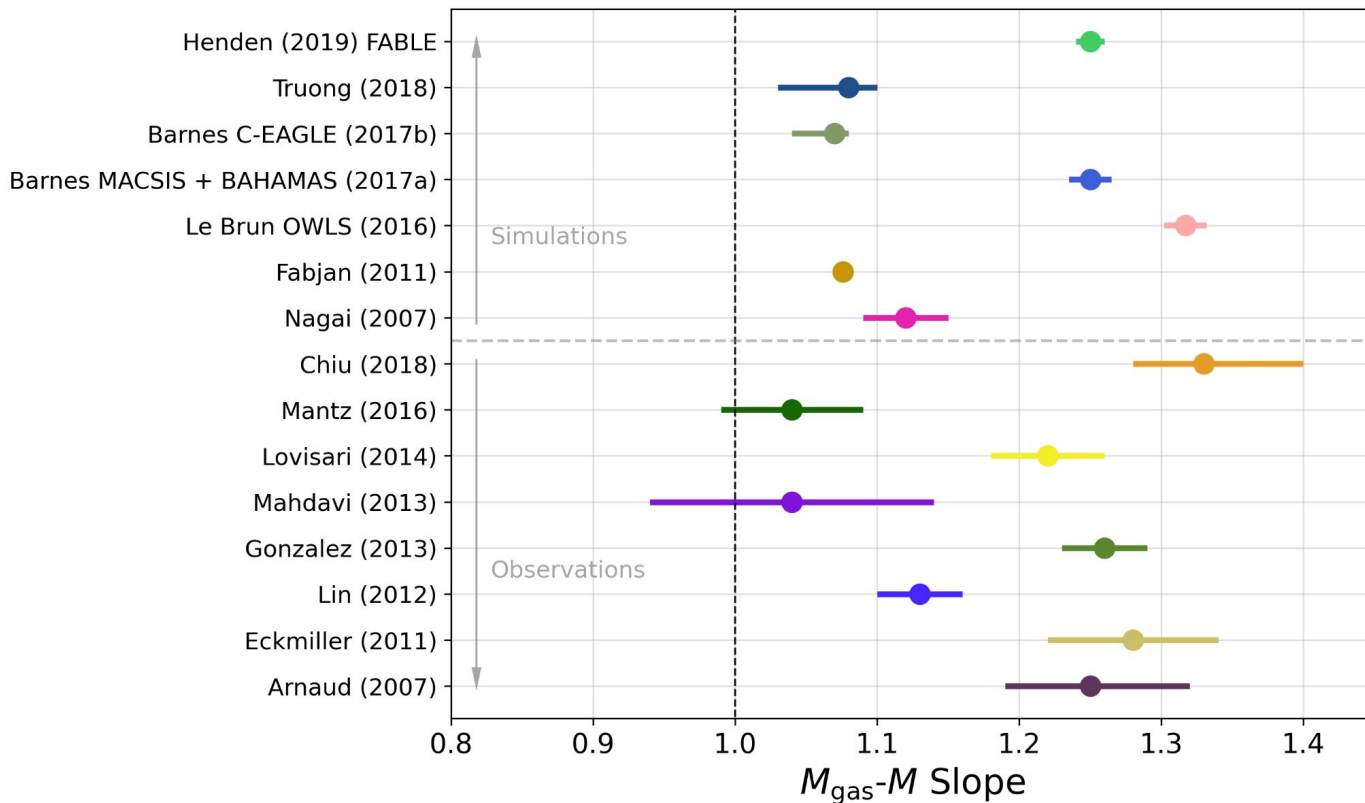
– Approaches self-similarity (5/3) @high masses

– Group slope steepens with time

# Compendium of literature slopes: gas mass (incomplete)



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**Super-linear** due to less efficient star formation in high mass halos.

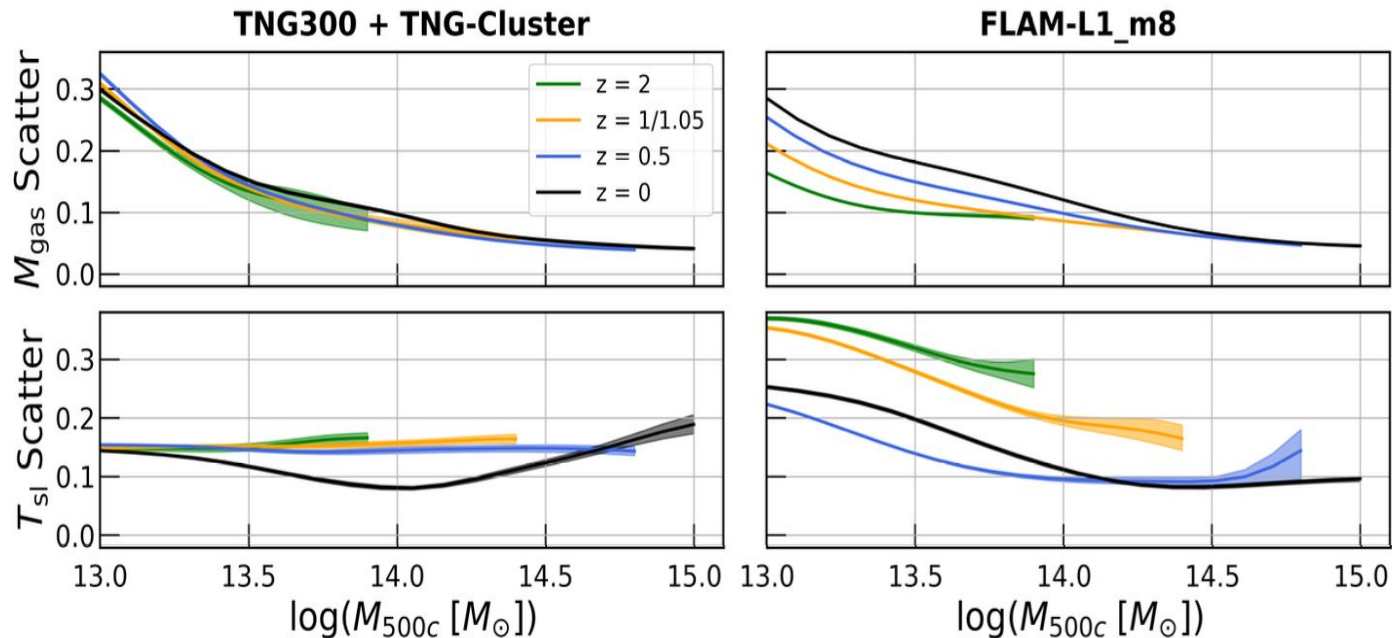
Good consistency but does not reflect

- sample selection/ mass coverage
- systematic errors

# Hot gas scaling relations: gas mass & temp. scatter



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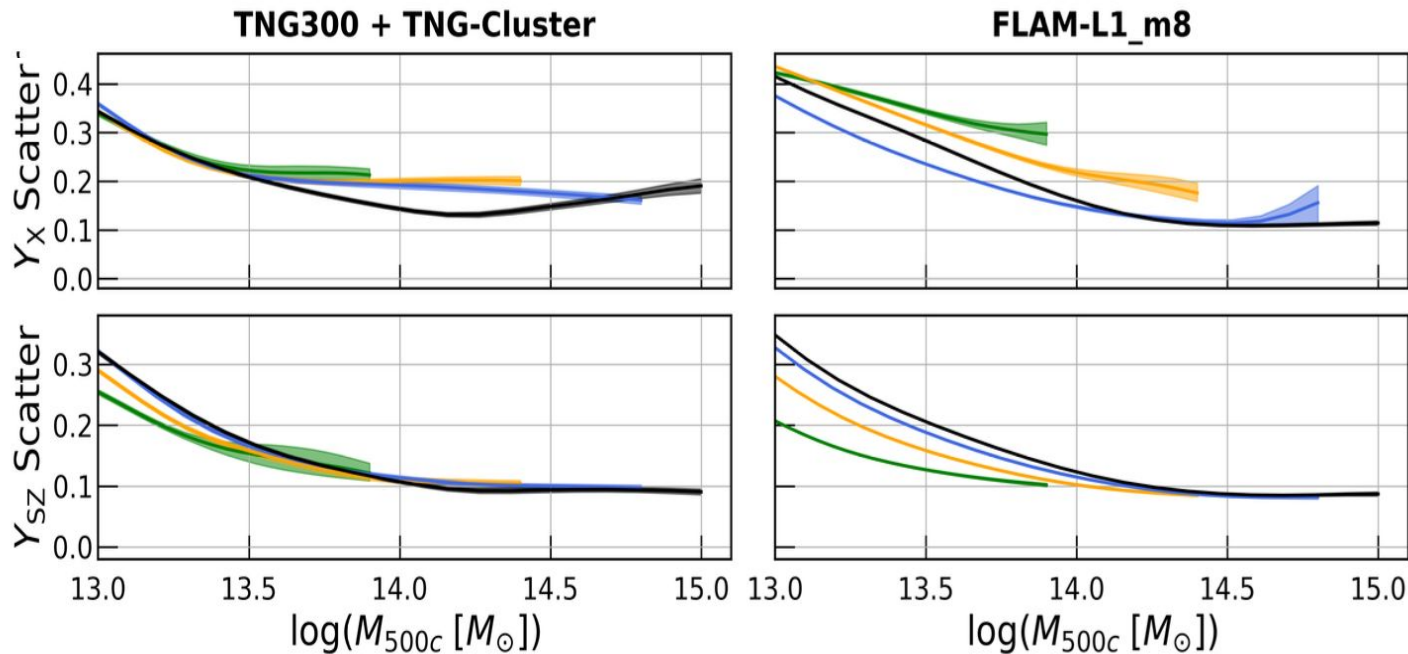
Strong mass dependence in  $M_{\text{gas}}$   
Only 5% @  $1e15$

More z-evolution in FLAM than TNG at the group scale.

# Hot gas scaling relations: pressure scatter



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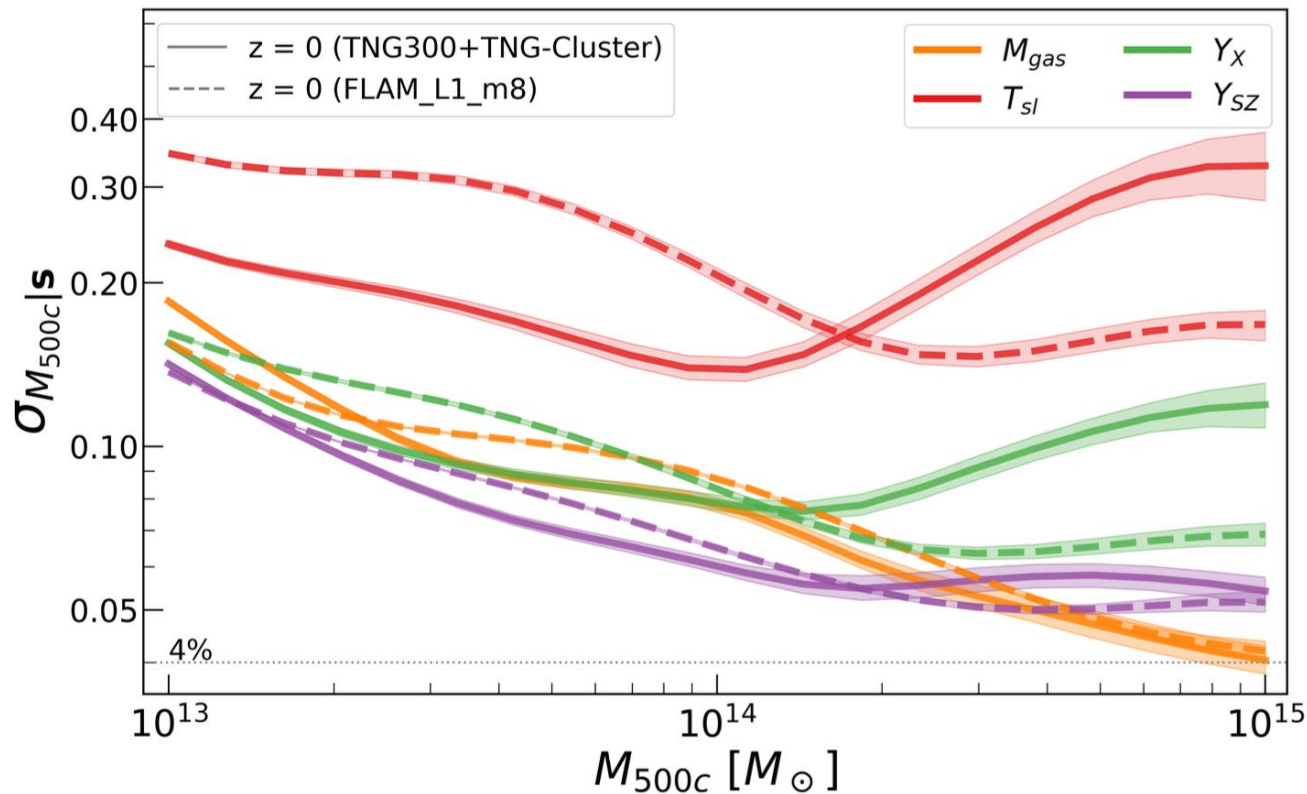


Asymptotes to 10%  
at cluster masses  
( $>1e14$ )

# Hot gas properties: Mass Proxy Quality @z=0



Eddie Aljamaal



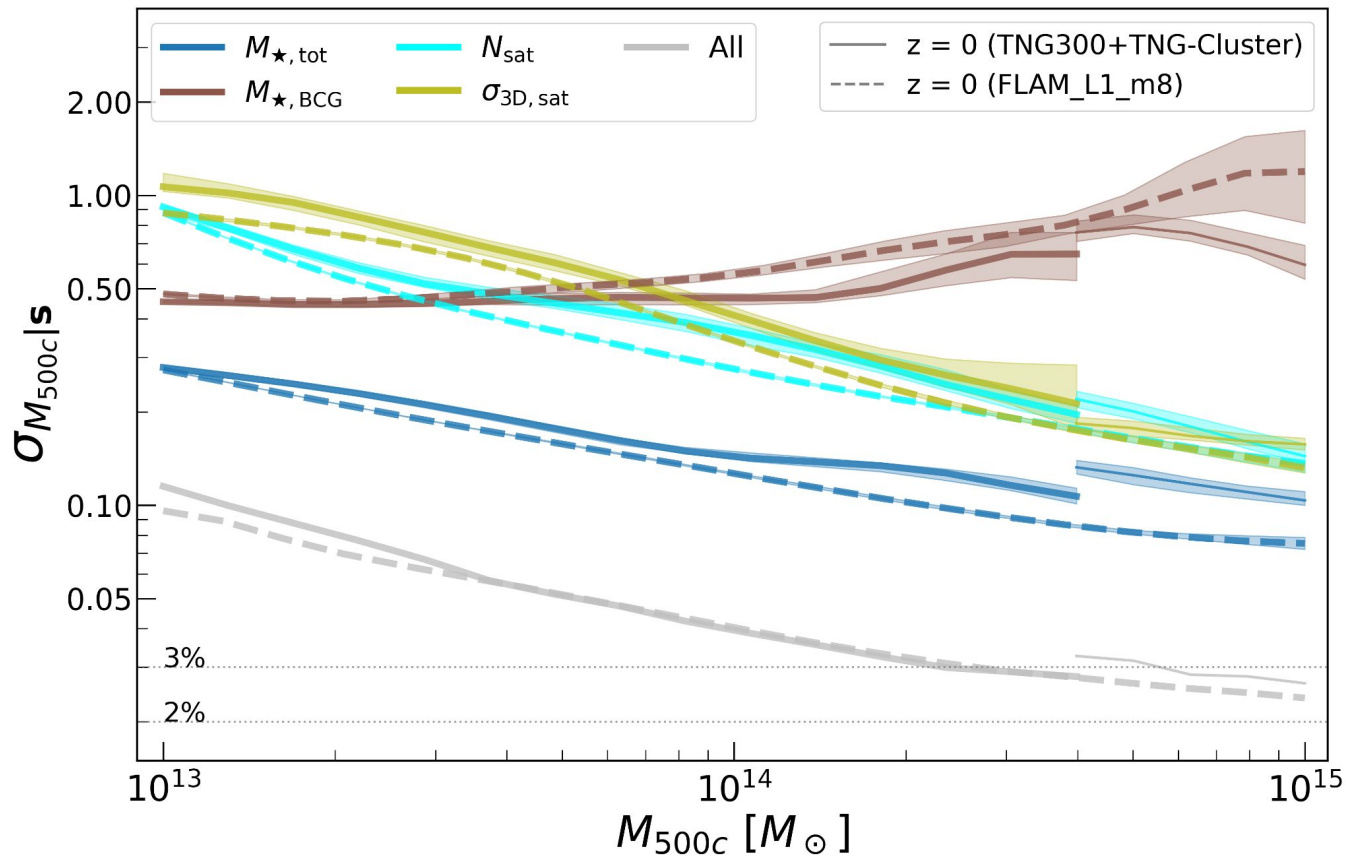
**Gas Mass** and **SZ Pressure** are the best proxies of halo mass.

Robust across redshift.

# What about Galaxy properties?



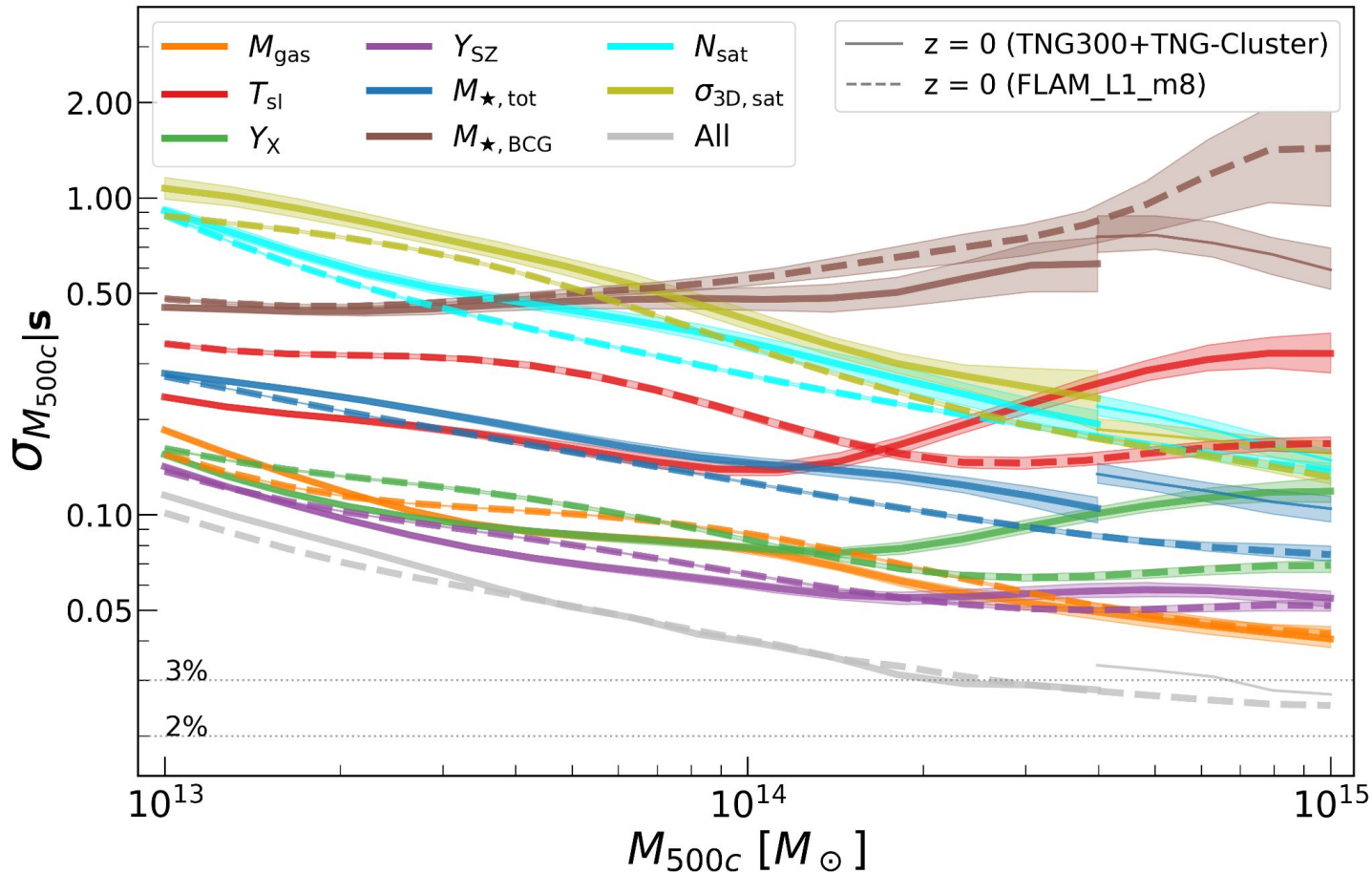
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**Total stellar mass is the best, reaching 10% for the largest clusters.**

**BCG stellar mass is relatively poor at high mass.**





**What if you had it all?**

**3% mass scatter is achieved at the cluster scale by combining all properties.**

**What can be achieved in practice?**

# Summary

Synergies between simulations and observations of galaxy clusters have been expanding and deepening for decades, and will continue to do so in the eRosita era.

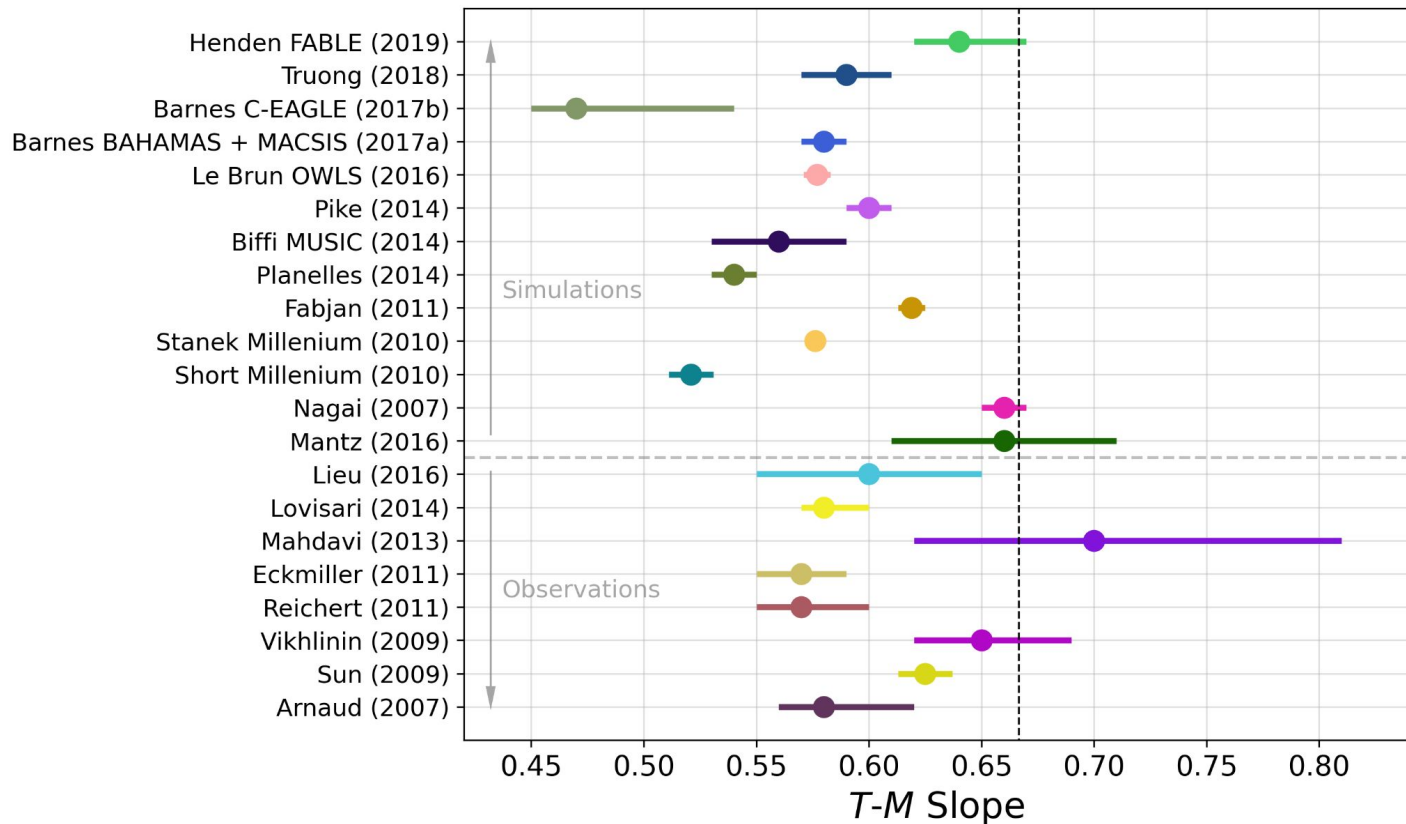
Multiwavelength studies are essential.

A verification study of two “full physics” cosmological simulations shows:

- Good agreement in hot gas property statistics,
- Mass and redshift dependence of slope and scatter, esp. at group scale,
- Gas mass and pressure are the best halo mass proxies.

**Thank you!**

# Compendium of literature slopes: Temperature

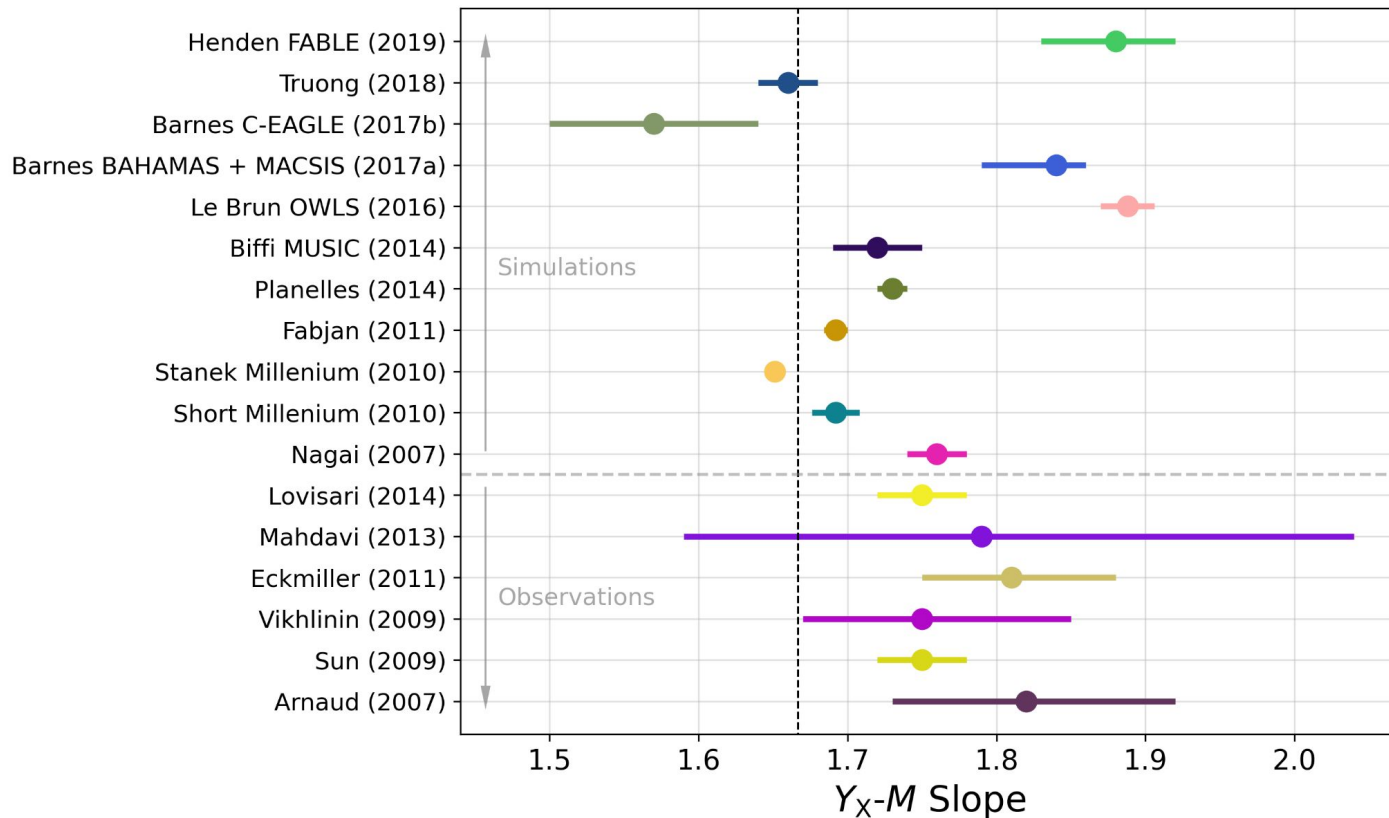


**Sub-self-similar** due to more efficient star formation in low mass halos.

Fair consistency but does not reflect

- sample selection
- systematic errors

# Compendium of literature slopes: Electron Pressure



**Super-self-similar**  
due primarily to  $M_{\text{gas}}$   
slope  $> 1$ .

Fair consistency but  
does not reflect  
– sample selection  
– systematic errors

# Property covariance: Mstarttot and Mgas

