Calorimeter-less gamma-ray telescopes: Optimal measurement of charged particle momentum from multiple scattering by Bayesian analysis of Kalman filtering innovations

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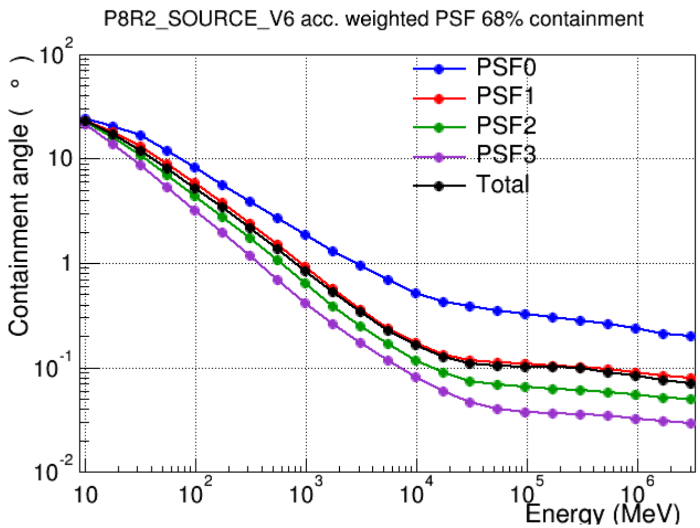
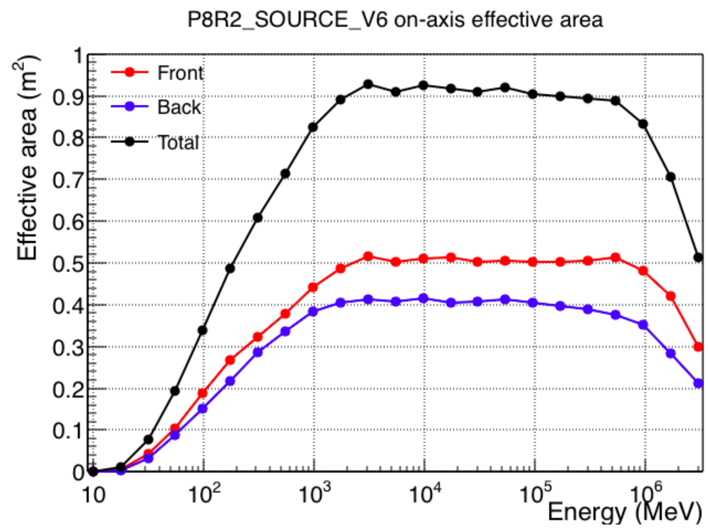
Mikael Frosini & Denis Bernard, Nucl. Instrum. Meth. A **867** (2017) 182, arXiv:1706.05863

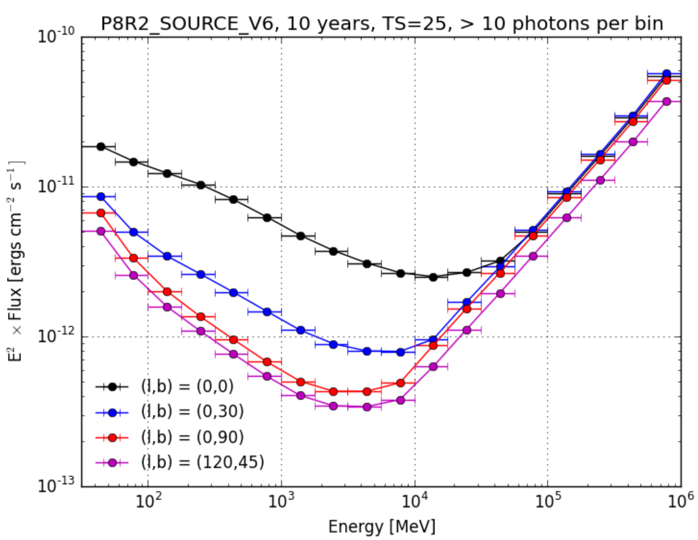
http://llr.in2p3.fr/∼dbernard/polar/harpo-t-p.html

W-slab Converter $\gamma \rightarrow e^+e^-$ *Telescopes*

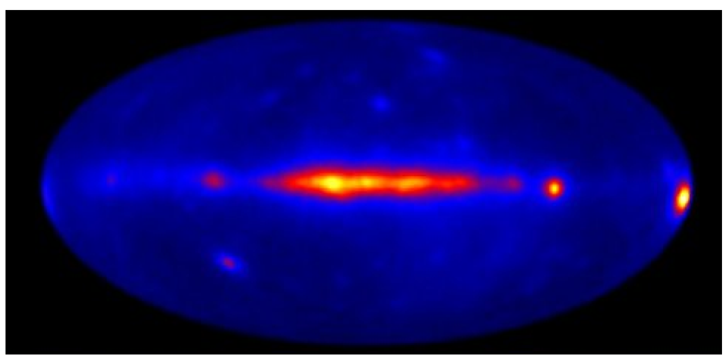
Fermi LAT Performance, Pass 8 Release 2 Version 6







32-56 MeV

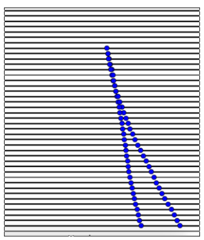


“Fermi-LAT below 100 MeV (Pass8 data)”, J. McEnery,
“e-ASTROGAM workshop, Padova 2017

High-Angular Resolution $\gamma \rightarrow e^+e^-$ *Telescope Projects*

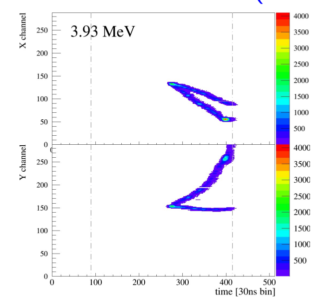
- Lower Z and/or lower density, higher spatial resolution

W-less, Si-stack detectors
AMEGO, e-ASTROGAM
1.3°@ 100 MeV
A. De Angelis *et al.*, Exp. Astr. **44** (2017) 25



Emulsions
GRAINE
1°@ 100 MeV
S. Takahashi *et al.*, PTEP **2015** (2015) 043H01

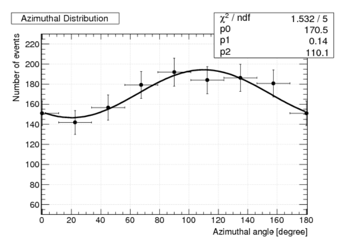
Gas time projection chamber (TPC)
HARPO
0.4°@ 100 MeV
D. Bernard, NIM. A 701 (2013) 225



Polarimetry with $\gamma \rightarrow e^+e^-$:

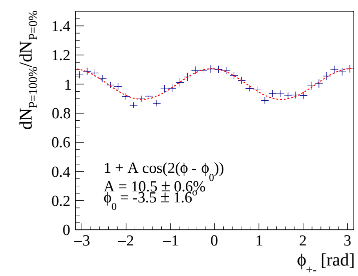
?

K. Ozaki *et al.*, NIM. A **833** (2016)165



GeV (50 MeV threshold ?)

P. Gros *et al.*, arXiv:1706.06483, Astropart. Phys.



MeV

- Lower effective area - per - tracker—converter mass

⇒ Larger volume ? ⇒ Calorimeter mass budget ? ⇒ γ-energy measurement ?

⇒ **charged-track momentum measurement ?**

Multiple Scattering & Track Momentum Measurement

Theory of the Scattering of Fast-Charged Particles (G. Molière):

- Fast charged particles deflected by detector ion and electron electric field

  I. Single Scattering on the Shielded Coulomb Field, Z.Naturforsch. A2 (1947) 133

- These small deflections average to a Gaussian angle distribution

  II. Plural And Multiple Scattering, Z.Naturforsch. A3 (1948) 78

  In modern form: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)

$$\theta_{RMS} = \frac{p_0}{\beta c p}\sqrt{\frac{x}{X_0}}\left(1 + \epsilon \log \frac{x}{X_0}\right)$$

  - p track momentum, $p_0 = 13.6\,\mathrm{MeV}/c$ "multiple scattering constant"
  - x path length and X_0 scatterer radiation length (cm)

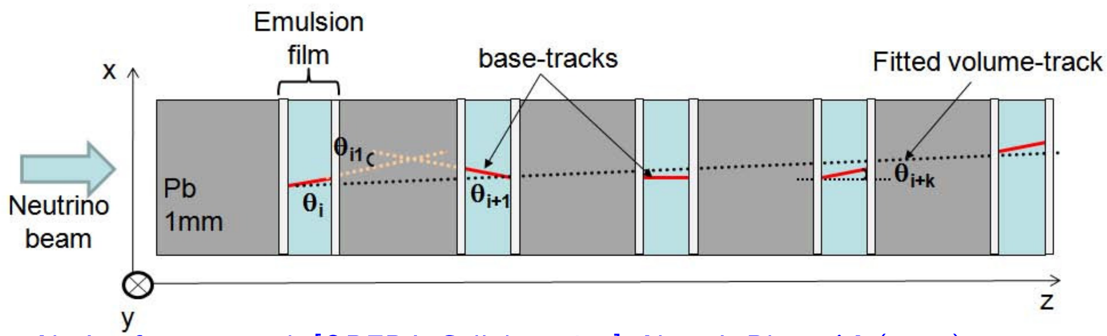
- **Measuring the statistics of these deflections yields a momentum measurement**

  III. The multiple scattering of traces of tracks under consideration of the statistical coupling, Z.Naturforsch. A 10 (1955) 177.

Momentum Measurement: Molière Method

Widely used for decades:

- Emulsion trackers
- Muon detector
  - Sampling detectors (instrumented Fe or Pb layer stacks)
  - lAr (liquid Argon) kilo-ton time projection chambers (TPC) for neutrino studies.
- ...



N. Agafonova *et al.* [OPERA Collaboration], New J. Phys. **14** (2012) 013026

- Track segment into tracklets, tracklet angle measured, deflection angle between tracklets n and $n+1$ computed.
- **How decide the segment length ?**

Thin Detectors: Segment Length Optimisation

- Relative momentum RMS resolution:

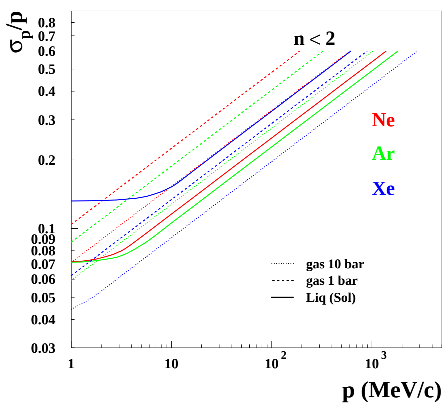
$$\frac{\sigma_p}{p} = \frac{1}{\sqrt{2L}} \left[\Delta^{1/2} + \frac{p^2 \sigma^2 X_0}{\Delta^{5/2} p_0^2} \right],$$

- Δ segment length, σ single track measurement spatial precision, L total track length. Minimum for:

$$\Delta = \left[\frac{5 p^2 \sigma^2 X_0}{p_0^2} \right]^{1/3}.$$

- **Optimal segmentation depends on momentum to be measured !**
- The value of σ_p/p for that optimal set is:

$$\frac{\sigma_p}{p} = \frac{C}{\sqrt{2L}} \left[\frac{p}{p_0} \right]^{1/3} \left[\sigma^2 X_0 \right]^{1/6}, \qquad C \equiv 5^{1/6} + 5^{-5/6} \approx 1.57.$$



Relative track momentum resolution for optimal sampling Δ as a function of track momentum. (Argon TPC)

D. Bernard, NIM. A 701 (2013) 225

Towards an Optimal Method ?

- How extract optimally all the multiple scattering information present in the track given the single track measurement spatial precision ?

- Kalman-filter tracking yields an optimal estimate of the track parameters (eg., lateral position and track angle at $z = 0$)

- Optimal treatment of **multiple scattering** and **single track measurement spatial precision** (Gaussian approximation)

- Kalman filter tracking yields an optimal estimate of the track parameters at point $n + 1$ from an optimal combination of
  - an optimal estimate of the track parameters at point n
  - the measurement at point $n + 1$

R. Frühwirth Nucl. Instrum. Meth. A **262**, 444 (1987).

- The update of the track parameters and of their covariance matrix involves
  - the track covariance matrix at n
  - the measurement covariance matrix (spatial resolution)
  - the process noise covariance matrix (multiple scattering, hence **track momentum !**

Optimal Method

- z_n, measurements
- x_n^{n-1}, prediction (at n, from the analysis of the points from 0 up to $n-1$)
- $\nu_n \equiv z_n - x_n^{n-1}$, innovations

  ν_n are Gaussian distributed, $\mathcal{N}(0, S_n(s))$

  $S_n(s)$, covariance matrix is computed while the Kalman filter, given s, is proceeding along the track

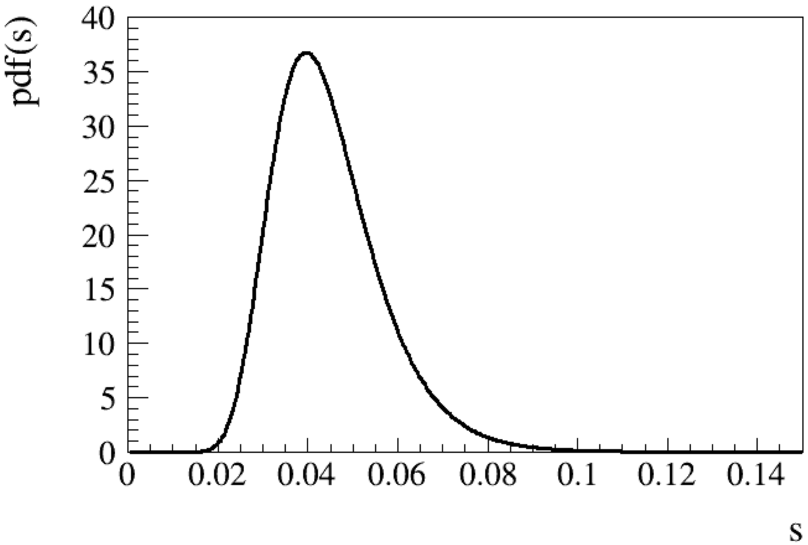
  $s \equiv \left(\frac{p_0}{p}\right)^2 \frac{\Delta x}{l X_0}$ is the average multiple-scattering angle variance per unit track length, $\theta_0^2 = s \times l$.

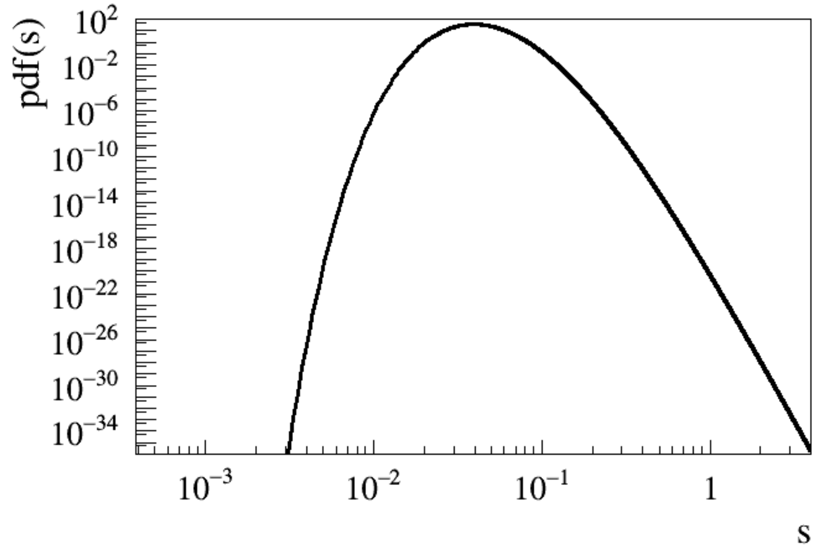
  l longitudinal sampling, Δx scatterer thickness. Homogeneous detector: $l = \Delta x$.

- $p_n(s)$ probability to observe such a track up to n, given s
- It can be shown that $p_n(s) \propto \prod_i \mathcal{N}(\nu_i(s), 0, S_i(s))$

P. Matisko and V. Havlena, Int. J. Adapt. Control Signal Process. 27 (2013) 957

Optimal Method: Results





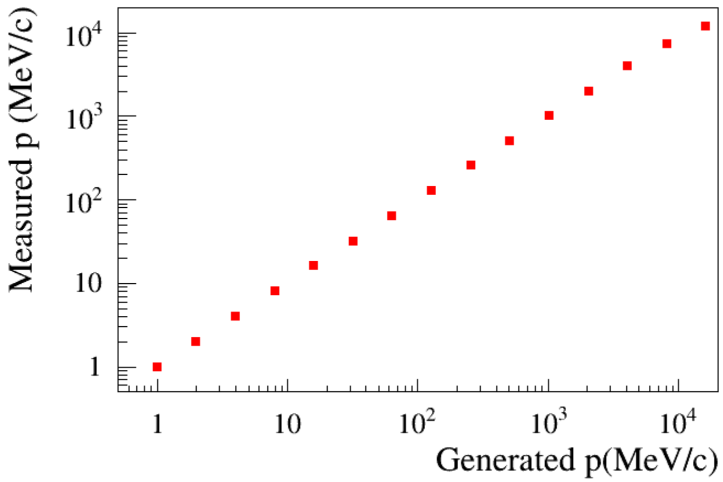
$p_N(s)$ distribution for a $50\,\mathrm{MeV}/c$ track in a silicon detector

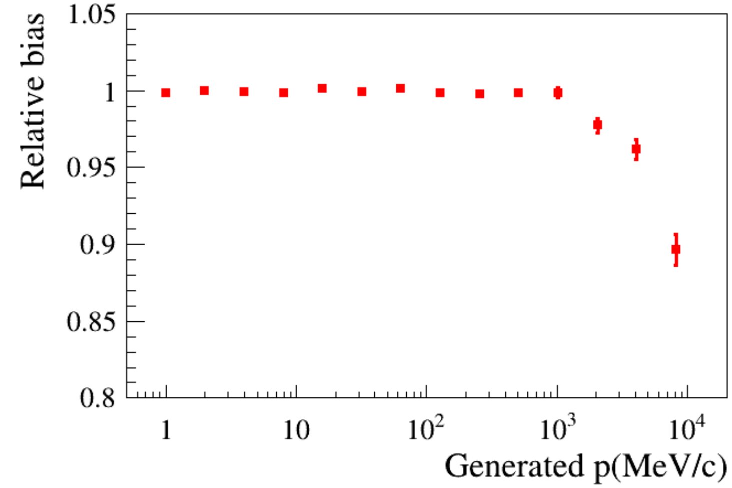
X_0	9.4	cm
l	1.0	cm
Δx	0.0500	cm
σ	0.0070	cm
N	56	

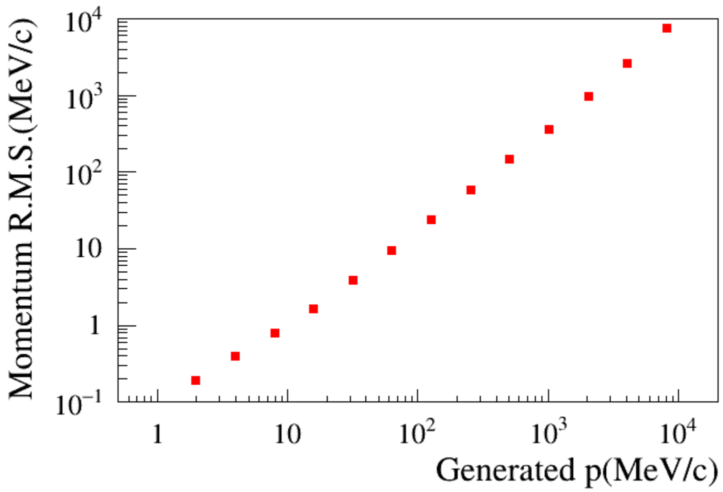
Obtain s that maximises $p_N(s)$ and $p = p_0 \sqrt{\dfrac{\Delta x}{l X_0 s}}$

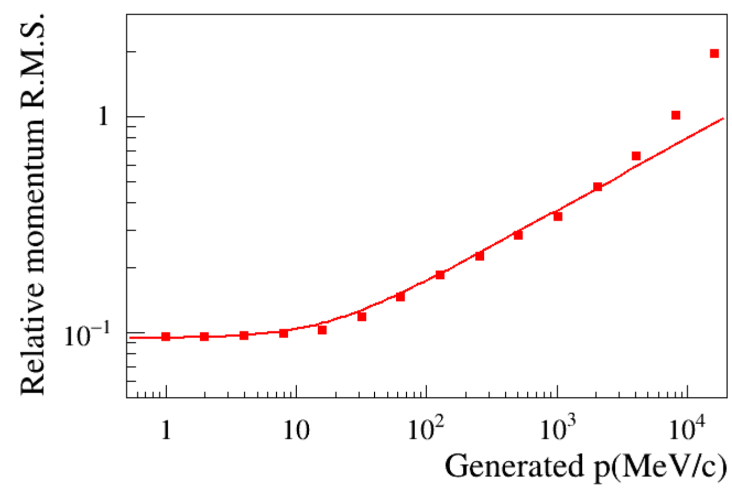
M. Frosini & D. Bernard, Nucl. Instrum. Meth. A **867** (2017) 182,

Performances









Silicon detector. 10^4 evts / MC sample.

Unbiased, usable up to a couple of GeV/c

M. Frosini & D. Bernard, Nucl. Instrum. Meth. A **867** (2017) 182,

Conclusion

Magnetic-field-free trackers :

- Molière method (1955): measures charged-track momentum from multiple measurements of multiple-scattering-induced deflections
- Kalman-filter track fit (Frühwirth, 1987): yields optimal unbiased charged-track parameters <u>when the momentum is known.</u>
- A Bayesian analysis of the filtering innovations of s-indexed Kalman filters yields an optimal, unbiased, estimate of the momentum (Frosini & Bernard 2017)
  - and of the other track parameters, BTW.

- Caveat: a number of approximations:
  - no energy loss in detector
  - no radiation
  - Gaussian multiple-scattering angle distribution and space resolution
  - ...

Back-up Slides

Parametrisation of the relative momentum resolution

A good representation of these data

$$\frac{\sigma p}{p} \approx \frac{1}{\sqrt{2N}} \sqrt[4]{1 + 256 \left(\frac{p}{p_0}\right)^{4/3} \left(\frac{\sigma^2 X_0}{N \Delta x\ l^2}\right)^{2/3}},$$

Low-momentum asymptote

$$\frac{\sigma p}{p} \approx \frac{1}{\sqrt{2N}}$$

High-momentum asymptote

$$\frac{\sigma p}{p} \approx \sqrt{\frac{8}{N}} \left(\frac{p}{p_0}\right)^{1/3} \left(\frac{\sigma^2 X_0}{N \Delta x\ l^2}\right)^{1/6}.$$

p_s, the momentum above which σ_p/p starts to depart from the low momentum asymptote,

$$p_s = p_0 \frac{1}{64} \left(\frac{N \Delta x\ l^2}{\sigma^2 X_0}\right)^{1/2}.$$

p_l, the momentum above which σ_p/p is larger than unity (meaningless measurement)

$$p_\ell = p_0 \left(\frac{N}{8}\right)^{3/2} \left(\frac{N \Delta x\ l^2}{\sigma^2 X_0}\right)^{1/2}.$$

Note that

$$p_\ell = p_s \, (2N)^{3/2}.$$

And

$$\frac{\sigma p}{p} \approx \frac{1}{\sqrt{2N}} \sqrt[4]{1 + \left(\frac{p}{p_s}\right)^{4/3}}$$

M. Frosini & D. Bernard, Nucl. Instrum. Meth. A **867** (2017) 182,

Parametrisation of the relative momentum resolution

High-momentum asymptote

$$\frac{\sigma_p}{p} \approx \sqrt{\frac{8}{N}} \left(\frac{p}{p_0}\right)^{1/3} \left(\frac{\sigma^2 X_0}{N \Delta x \; l^2}\right)^{1/6} \qquad L = l \times N$$

$$\frac{\sigma_p}{p} \approx \sqrt{8} \left(\frac{p}{p_0}\right)^{1/3} \left(\frac{\sigma^2 X_0}{N^2 \Delta x \; L^2}\right)^{1/6} \approx \sqrt{8} \left(\frac{p}{p_0}\right)^{1/3} \left(\frac{\sigma}{N L}\right)^{1/3} \left(\frac{X_0}{\Delta x}\right)^{1/6}.$$

- For a given **wafer** thickness, Δx, and total detector thickness L, improvement with larger N (smaller l)
  - part of it by improving the precision of the position over a given segment length $\frac{\sigma}{\sqrt{N}}$
  - the rest of it, $\left(\frac{1}{\sqrt{N}}\right)^{1/3}$, use of short scale multiple scattering information
- For **homogeneous active targets**, $\Delta x = l$ (gas TPC telescopes, lAr TPC ν detectors)

  $$\frac{\sigma_p}{p} \approx \sqrt{8} \left(\frac{p}{p_0}\right)^{1/3} \left(\frac{\sigma^2 X_0}{N L^3}\right)^{1/6}$$

  i.e., the same residual $\left(\frac{1}{\sqrt{N}}\right)^{1/3}$, scaling